Meas. Sci. Technol. **10** (1999) 454–459. Printed in the UK PII: S0957-0233(99)01412-5

A rotating torsion balance experiment to measure Newton's constant

J H Gundlach

Department of Physics, Nuclear Physics Laboratory, University of Washington, Seattle, Washington 98195, USA

Received 28 January 1999, in final form and accepted for publication 8 March 1999

Abstract. We have developed a new method to determine the gravitational constant. This method severely reduces the sensitivity to the leading sources of error in previous torsion balance measurements: it is mostly free of uncertainties due to anelastic torsion fibre properties. The torsion pendulum mass density distribution does not have to be known accurately. Gravitational effects due to mass distribution changes in the vicinity are reduced. The apparatus is currently under construction.

Keywords: gravity, Newton's constant, precision measurement, torsion balance

1. Introduction

Based on our development of rotating torsion balances for equivalence principle tests [1] we have invented a method to measure the gravitational constant G. This work was stimulated by the results of recent measurements that varied widely and the relatively large uncertainty in the accepted value of this fundamental constant [2–6]. Our method is designed to eliminate or reduce the leading uncertainties of previous torsion balance measurements and should therefore yield a more precise and reliable value of G. In the most precise measurement carried out to date [7] the leading uncertainty stems from the mass distribution of the pendulum. We discovered that by only requiring the pendulum to be a thin flat vertical plate one can make a torsion balance measurement independent of the shape, mass distribution and mass of the pendulum. Arranging the attractor masses in a special configuration enhances this independence. Another cause of uncertainty in conventional torsion balance measurements is due to the torsion fibre properties. Kuroda pointed out [8] and Bagley and Luther demonstrated that a systematic bias in the measurement could be due to anelastic properties of the torsion fibre [9]. In our method we avoid these problems since the torsion fibre will practically not be twisted.

Statistical noise due to a changing gravitational field in the vicinity of the apparatus or due to torsion fibre noise will be reduced since the frequency at which the signal is extracted can be freely selected and made high to reduce noise. We have tested several aspects of our method with simulations and experimental tests. Our apparatus is currently under construction. We expect that metrology on the attractor masses will be the most dominant uncertainty in our measurement.

2. The new method

2.1. Schematic description

The following is a highly simplified description of our new method. We place a torsion balance on a slowly and continuously rotating turntable located between two attractor masses (spheres). The torsion fibre is then twisted due to the gravitational attraction on the pendulum. This twist is sensed and a feedback is turned on that changes the turntable rotation rate to minimize the twist. Therefore the angular acceleration of the turntable is made equal to the gravitational angular acceleration on the pendulum. It is measured directly with a high-resolution shaft encoder and a clock. The attractor masses are placed on a separate turntable that rotates at a constant rate and independently of the inner turntable. The angular acceleration of interest occurs at $\sin 2\phi$, where ϕ is the angle between the pendulum and the attractor masses.

2.2. The gravitational angular acceleration, the flat plate pendulum advantage

The gravitational angular acceleration about the torsion axis of a torsion pendulum caused by the field of an external mass distribution can be written as a multipole expansion:

$$\alpha = \sum_{l,m} \alpha_{lm} = -\frac{4\pi G}{I} \sum_{l=2}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{+l} m q_{lm} Q_{lm}\, \mathrm{e}^{\mathrm{i}m\phi}. \quad (1)$$

Here q_{lm} and Q_{lm} are the mass multipole moments of the pendulum and the multipole fields of the external mass distribution, respectively, as given below. ϕ is the azimuthal angle of the pendulum with respect to the mass distribution and I is the moment of inertia about the torsion fibre.

$$q_{lm} = \int \rho(\vec{r}_p) Y_{lm}(\theta_p, \Phi_p) r_p^l\, \mathrm{d}^3 r_p \quad (2)$$

0957-0233/99/060454+06$19.50

$$Q_{lm} = \int \rho(\vec{r_a}) Y_{lm}(\theta_a, \Phi_a) r_a^{-l-1}\, \mathrm{d}^3 r_a. \qquad (3)$$

Here $\rho(\vec{r_p})$ and $\rho(\vec{r_a})$ are the mass distributions of the pendulum and of the attractor respectively.

For most Cavendish-type experiments the term due to $l = 2$ and $m = 2$ is by far the biggest term in equation (1).

$$\alpha(\phi) \approx \alpha_{22} = -\frac{16\pi}{5} G \frac{q_{22}}{I} Q_{22} \sin 2\phi. \qquad (4)$$

Closer inspection of the quotient q_{22}/I reveals that in the limit of the pendulum being a two-dimensional plate the shape, total mass, mass distribution, etc cancels and the quotient becomes a constant:

$$\frac{q_{22}}{I} = \frac{\int \rho(\vec{r_p}) Y_{22}(\theta_p, \phi_p) r_p^2\, \mathrm{d}^3 r_p}{\int \rho(\vec{r_p}) \sin^2\theta_p\, r_p^2\, \mathrm{d}^3 r_p} \xrightarrow{\text{2D}} \sqrt{\frac{15}{32\pi}} \qquad (5)$$

and therefore the biggest contribution to the angular acceleration is:

$$\alpha_{22} = -\sqrt{\frac{24\pi}{5}} G\, Q_{22} \sin 2\phi. \qquad (6)$$

This remarkable cancellation has not been explicitly used in any G torsion balance measurement. In the currently still most precise determination of G by Luther and Towler the uncertainty due the pendulum mass distribution was the largest single source of uncertainty [7].

A realistic pendulum must have a finite thickness t. This leads to a small correction to the ideal ratio (equation 5):

$$\frac{q_{22}}{I} = \frac{w^2 - t^2}{w^2 + t^2} \sqrt{\frac{15}{32\pi}}. \qquad (7)$$

For our pendulum body (described below) this will be a 7.8×10^{-4} correction and the overall uncertainty in G is only relatively weakly affected by the uncertainty in pendulum thickness, flatness and width (also see table 1).

We will harvest the full advantage of the q_{22}/I cancellation by eliminating all significant multipole angular accelerations other than α_{22}. An up–down symmetric pendulum and a symmetric attractor mass distribution will be used so that the only remaining angular acceleration contributions are: α_{42}, α_{44} and α_{62}, α_{64}, α_{66}, etc. The magnitudes of the higher order accelerations α_{lm} diminish quickly as the series converges as $(R_{pend}/R_{attr})^{l-2}$. Here R_{pend} and R_{attr} are the typical radii for the pendulum and the attractor mass distribution. Angular accelerations with $m = 2$ have to be accounted for carefully since they occur at the same frequency as the main angular acceleration of interest, α_{22}. Using a simple rectangular pendulum plate with the height h related to the width w and thickness t by

$$h^2 = \tfrac{3}{10}(w^2 + t^2) \qquad (8)$$

eliminates the q_{42} moment. In addition the Q_{42} of the attractor can be designed to be zero: instead of selecting only one attractor sphere per side two spheres on each side are used. They are spaced vertically apart by: $z = \sqrt{2/3}\rho$ where ρ is the radial distance from the pendulum axis. The angular acceleration α_{42} is therefore doubly eliminated by $q_{42} = 0$ and $Q_{42} = 0$. We also chose to eliminate α_{44} even though it does not occur at $\sin 2\phi$: this is done by using two pairs of spheres on each side that are 45° of azimuth apart.

Table 1. Approximate proposed 1σ error budget. For the position error it was assumed that the average of four different configurations is taken. The statistical error corresponds to ten days of data acquisition. A large uncertainty could be due to a systematic offset in the distance measurement. Improperly accounted surface roughness of the attractors or miscalibration of the distance measuring tool could have such an effect.

Quantity	Estimated measurement precision	$\Delta G/G$ (ppm)
Systematic errors		
Pendulum:		
height	10 μm	0.0
width	10 μm	0.2
thickness	2.5 μm	2.5
tilt	10 μrad	0.5
mass	10 μg	0.0
density	0.2%	0.5
Attractor masses:		
pair distances		
diagonal	2.5 μm	2.5
adjacent	2.5 μm	5.0
vertical	2.5 μm	3.3
mass	10 mg	1.3
centre of mass	1 μm	2.0
Turntable alignment:		
radial	0.1 mm	0.03
tilt	0.1 mrad	0.0
height	0.1 mm	0.001
Distance measuring		
offset:	0.5 μm	9.4
Twist angle detector:	0.1%	1.0
Time base:	10^{-8}	0.01
Statistical errors:		
Torsion balance	$<7.5 \times 10^{-12}\sqrt{\mathrm{day}}/\mathrm{s}^2$	3
Q_{22} fluctuations	$<10^{-5}\sqrt{\mathrm{day}}$	3
Total		≈ 12.7

Higher α_{l2} are small and calculable. For example

$$\frac{\alpha_{6,2}}{\alpha_{2,2}} = \frac{99}{7683200} \frac{213(w^4 + t^4) + 626w^2t^2}{\rho^4} \qquad (9)$$

is only a small, 1.2×10^{-4}, contribution to the total acceleration for our set-up and therefore does not require exact knowledge of the pendulum mass distribution.

2.3. Angular acceleration readout and feedback

The feedback method forces the turntable to closely follow the torsion pendulum's gravitational angular acceleration. The method has several significant advantages: the torsion fibre practically does not twist. This eliminates problems with anelasticity or inelasticity which may have caused problems in many previous measurements [8–10]. The readout method is inherently extremely linear. The acceleration is directly read with a high-resolution angular encoder that is mounted on the turntable. The angle calibration is automatic since $0^\circ = 360^\circ$ and the continuous rotation averages over the entire angular encoder avoiding problems with a possible local nonlinearity. A cross calibration using electrostatic methods is not required. The pendulum spring constant is reduced by the feedback gain

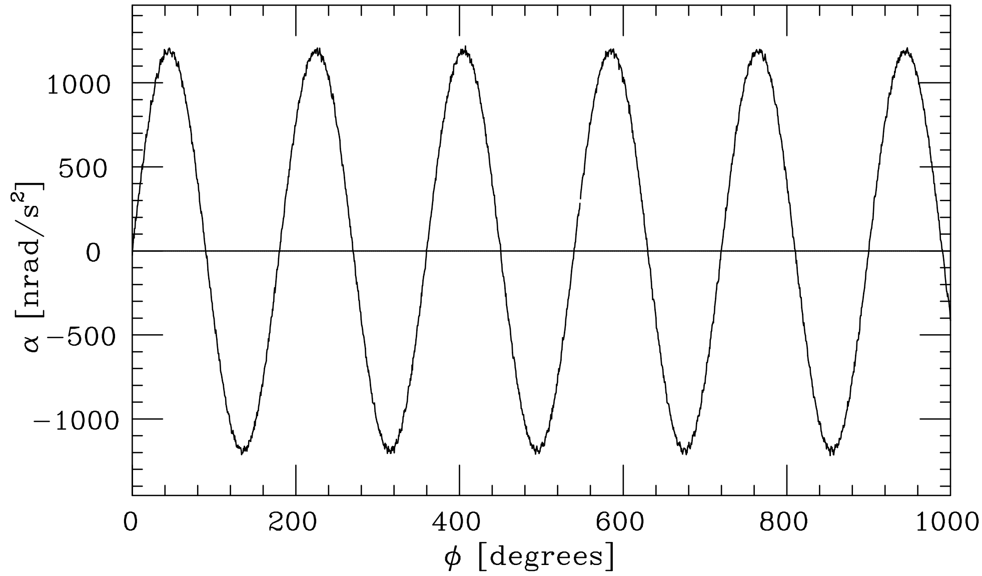

Figure 1. Angular acceleration from a numerical simulation of the measurement. These simulations were made to find the best feedback algorithm, to test the precision of our data analysis in recovering the input value of G and to estimate the statistical uncertainty in G.

making the pendulum quasi-free. The measurement can therefore be taken at any reasonable average turntable rotation rate. The signal extraction frequency can be chosen by the operator and can, e.g., be a relatively high frequency which is advantageous to reduce $1/f$ noise inherent in torsion fibres. Dynamic effects due to resonant amplitude build-up are strongly reduced.

The signal of interest is derived offline by forming the numerical second derivative with respect to time of the turntable angle and fitting it to $\sin 2\phi$. This cleanly isolates the torques with $m = 2$.

Any feedback scheme requires that a small twist remains in the torsion fibre to derive a feedback signal. The torque tied up in this small twist slightly reduces the measured angular acceleration. The twist angle is recorded and the missed acceleration will be added back. We will therefore have to know the twist angle calibration. For a 10^{-5} measurement and a gain of the feedback loop $>10^3$ the calibration has to be known to $<1\%$. We have the ability to perform quite accurate twist angle calibrations by turning the feedback off and performing rotation velocity step changes.

We have numerically tested several feedback loop configurations. We found excellent performance for a feedback function containing a proportional, integral and a differential (PID) term. In these simulations a gain of the loop of several thousand was obtained. The gain of the loop is defined as the total gravitational torque divided by the residual torque tied up in the torsion fibre twist. A much larger gain of $>10^5$ was reached in the simulations by using a learning mode in which the feedback function used speed change information from the previous revolutions. All our simulations included realistic $1/f$ fibre noise as well as white noise as typically seen in the pendulum angle readout. Delays due to the electronic time constants of the angle detection system and finite data taking intervals were also included. The simulated data analysis showed that the statistical noise permits one to measure G at the 10^{-5} level in less than one day of operation. The simulations allowed us to test our data analysis procedure and we were able to extract the value of G to within 10^{-6}. Figure 1 shows the angular acceleration signal obtained with our numerical simulations. The assumed Q_{22} source strength was comparable to the value for the actual apparatus.

We used an existing rotating torsion balance to test various aspects of our method and in particular the feedback scheme. An apparatus previously used for equivalence principle tests and as described elsewhere [1] was modified. A lab-fixed Pb attractor with $Q_{22} = 0.52$ g cm^{-3}, which is roughly a factor five smaller than the source for the G measurement, was used. Two of the normally four equivalence principle test testbodies were removed to create a sizable q_{22} moment. The average turntable speed was set to $\bar{\omega}_t = 0.0011$ rad s^{-1}, resulting in a 3.7% gravitational $\sin 2\phi$ speed variation. The feedback gain was $\approx$2000. The extracted value of G agreed surprisingly well with the standard value, but a 2% uncertainty mostly due to uncertainties in the attractor mass distribution had to be assigned. Figure 2 shows the measured acceleration of a run that took about 5 h.

2.4. The source rotation advantage

Experimental tests of our method showed that gravity gradient fluctuations caused by human activity in the building are a significant source of noise. We therefore expect that systematic uncertainties also arise due to human activities that change the environmental gravitational gradients during the measurement. To reduce both uncertainties we will continuously rotate the attractor masses on a separate but coaxial turntable. The acceleration signal will then be modulated at twice the difference between the inner and outer turntable angles. The attractor mass turntable velocity can be made fast without reducing the amplitude of the gravitational angular acceleration. Since cultural noise as well as the intrinsic torsion fibre noise have $1/f$ character this leads to an effective reduction in statistical noise. With the rotating

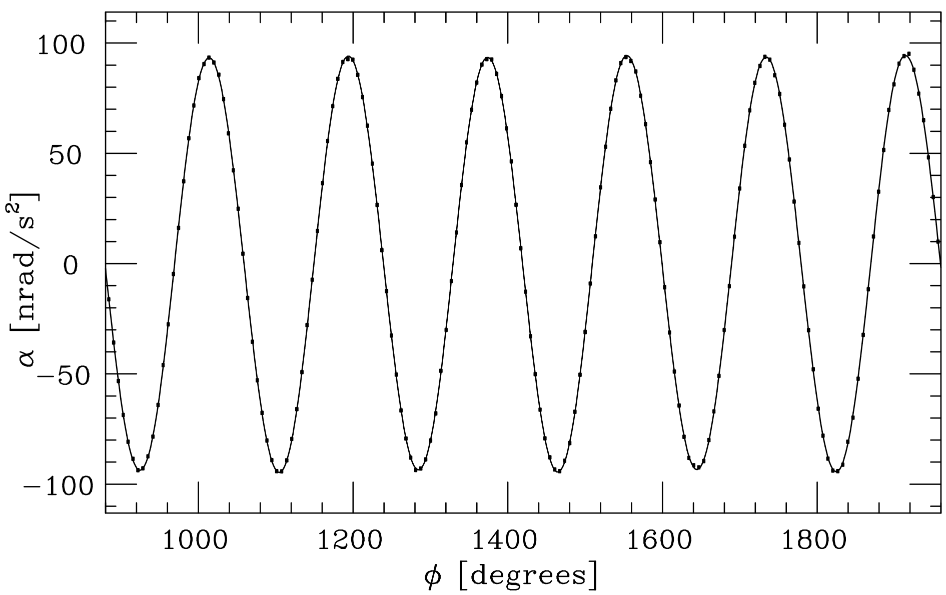

Figure 2. Proof-of-principle. We modified our existing rotating torsion balance to demonstrate the acceleration feedback method. Shown is the measured angular acceleration (averaged over 100 s) as a function of turntable angle. The solid line is a fit to the data. The attractor source mass in the real measurement will be five times stronger. Gravitational fluctuations from human activity in the vicinity of the balance were a significant source of noise.

attractor masses a lab-fixed mass will produce a signal at a frequency different from the signal frequency. Other systematic uncertainties associated with a tilt or wobble, or other imperfections of the torsion balance turntable, will not occur at the signal frequency either. Similarly, possible magnetic or paramagnetic couplings to lab-fixed fields become unimportant. In addition the rotation of the attractor masses presents another layer of immunity to possible angular encoder imperfections of the inner turntable.

3. The apparatus

An instrument that incorporates all the above features has been designed and is currently under construction. A cut-away view is shown in figure 3. Since the apparatus is under development technical details may be different in the actual final apparatus. The vacuum vessel in which the torsion pendulum hangs is mounted on an air bearing. A centre-less 36 000 line optical shaft encoder is direct mounted to the air bearing. Two readheads are installed 180° apart. A copper drag cup of an eddy current motor is attached to the air bearing. The eddy current motor stator is taken from a three-phase motor which is driven by three separate amplifiers. A 40 cm long, 17 μm diameter tungsten torsion fibre holds a 1.5 mm thick gold-coated glass plate of width $w = 76.2$ mm and $h = 41.7$ mm. The top of the torsion fibre is attached to an eddy current swing damper which in turn is attached to a rotation stage to align the pendulum with respect to the twist angle read out. A small 8 l s^{-1} ion pump maintains a 10^{-5} Pa vacuum. Inside the vacuum chamber is a μ-metal shield that surrounds the pendulum.

We use a novel technique to read out the pendulum twist angle (figure 4). The light from a laser diode is made into a 0.5 cm diameter parallel beam using a 30 cm focal length lens. This beam is reflected off the pendulum plate to three stationary flat mirrors that guide the beam to the back side of the plate. Another stationary flat mirror reflects the beam back on practically the same path bouncing it once more off the back side and the front side. It then passes through the same lens and half of the light is directed via a half-silvered mirror onto a position sensitive detector. This new scheme has two major advantages: it amplifies the torsion angle by eight due to the four bounces off the mirrors. This amplification is free of noise. Secondly, the system is only sensitive to rotations about the torsion fibre axis and is insensitive to rotations about any other axis.

The eight attractor masses are 125 mm diameter 316-stainless steel spheres weighing ≈8 kg each. The spheres are placed at a radius of 16.5 cm from the torsion fibre and height and azimuth as described above. Each sphere rests on three stainless steel seats which are mounted on the aluminium trays of the attractor turntable. The trays are supported by webs which rest on a 5 cm thick aluminium base that is mounted on a precision steel bearing. The attractor turntable is driven by a DC motor friction drive. The output pulse train of a 18 000 line/rev shaft encoder is used to derive a feedback that ensures a constant rotation rate. The spheres can be easily rotated to average out density fluctuations within the spheres. The spheres can be moved to positions 90° in azimuth apart on the turntable to discriminate against gravitational torques due to the turntable structure itself.

A digital signal processor (DSP) will be used to compute the feedback for the inner turntable and the attractor turntable. The DSP reads the turntable angles, the autocollimator angle, a clock signal and several other sensors and uploads the data to a host computer for storage.

The apparatus is located in the centre of an unused circular 12 m diameter cyclotron cave in the Nuclear Physics Laboratory on the campus of the University of Washington. The room is mostly underground and exhibits only small temperature fluctuations but a second actively controlled temperature environment will be built.

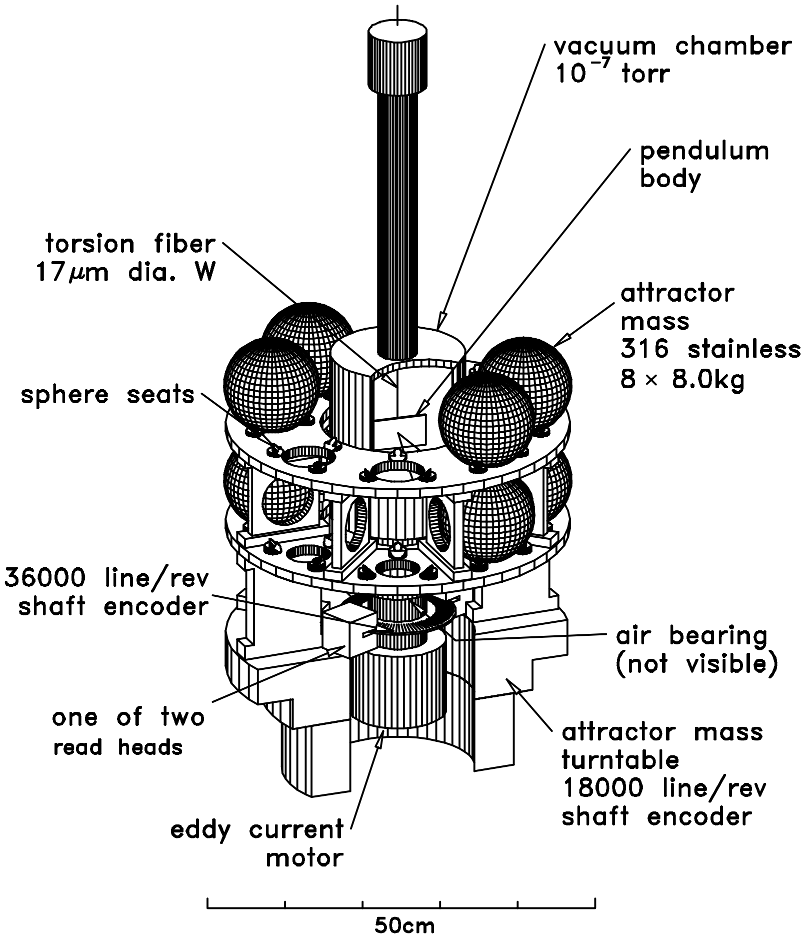

Figure 3. Cut-away view of the apparatus. The torsion balance, consisting of a flat vertical plate hung from a thin tungsten fibre, is mounted on an air bearing turntable. The apparatus rotates continuously and smoothly changes its angular velocity so that the torsion fibre is not twisted. The gravitational angular acceleration is transfered to the turntable and a high-resolution shaft encoder is used to read out the angular acceleration. Eight stainless steel masses produce an almost pure gravitational Q_{22} field. To eliminate gravitational accelerations from nearby masses, the attractor spheres are located on a second turntable and rotated with a constant speed in the opposite direction.

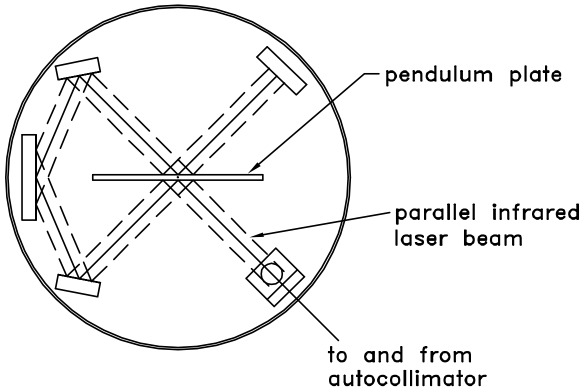

Figure 4. The pendulum deflection angle is read with an optical beam from an autocollimator system. The beam is reflected off the pendulum plate front side and is then guided by three mirrors to the back side. Another stationary mirror reflects the beam back on almost the same path into the autocollimator. The four reflections multiply the twist angle by a factor of eight without introducing extra noise. This system is only sensitive to rotations about the fibre axis.

4. Expected precision

With this measurement we hope to provide a reliable value for G at the $\approx 10^{-5}$ level. In table 1 we list our proposed error budget. The sensitivity to the pendulum mass distribution is dramatically reduced compared to traditional torsion balance experiments [7]. Exact knowledge of the attractor mass distribution becomes the leading source of uncertainty. Uncertainties due to variations in the density profile and sphericity of the spheres are effectively reduced by rotating the spheres between measurements. A miscalibration or systematically misread distance measuring tool is potentially the most severe source of error. The relative alignment of the torsion pendulum with respect to the attractor masses is not critical.

Statistical noise is due to the torsion balance and its readout and to fluctuations in the ambient gravitational field. To estimate the torsion balance noise we used the above described simulations and experimental tests. The noise estimate due to fluctuations in the laboratory Q_{22} field is based on typical people and car traffic in and around the lab.

5. Summary

A new method to measure G using a torsion balance on a turntable has been developed. The pendulum mass distribution, apart from being a flat vertical plane, does not have to be determined exactly. With the feedback scheme the torsion fibre inelasticity and anelasticity do not directly affect the measurement since the torsion fibre is not twisted. The attractor masses are also rotated but in a different speed and/or direction so that $1/f$ noise is reduced. These three features dramatically reduce the leading ambiguities of previous torsion balance measurements. We believe that the precise metrology of the attractor mass configuration will be the limiting source of uncertainty. The apparatus is currently being built. We finally hope to determine G to $\approx 10^{-5}$.

Acknowledgments

We specially thank NIST for the financial support of this work with precision measurement grant #60NANB7D0053. Further important support was provided through NSF grant #PHY-9602494, the Physics Department of the University of Washington, and the DOE supporting the Nuclear Physics Laboratory.

References

[1] Su Y, Heckel B R, Adelberger E G, Gundlach J H, Harris M, Smith G L and Swanson H E 1994 *Phys. Rev.* D **50** 3614
[2] Michaelis W, Haars H and Augustin R 1995 *Metrologia* **32** 267
[3] Fitzgerald M and Armstrong T R 1995 *IEEE Trans. Instrum. Meas.* **44** 494
[4] Walesch H, Meyer H, Piehl H and Schurr J 1995 *IEEE Trans. Instrum. Meas.* **44** 491
[5] Izmailov V P, Karagioz O V, Kuznetsov V A, Mel'nikov V N and Roslyakov A E 1993 *Meas. Tech.* **36** 1065
[6] Cohen E R and Taylor B N 1987 *Rev. Mod. Phys.* **59** 1121
[7] Luther G G and Towler W R 1982 *Phys. Rev. Lett.* **48** 121
[8] Kuroda K 1995 *Phys. Rev. Lett.* **75** 2796
[9] Bagley C H and Luther G G 1997 *Phys. Rev. Lett.* **78** 3047
[10] Matsumura S, Kanda N, Tomaru T and Kuroda K 1998 *Phys. Lett.* A **244** 4